Radiative large–angle Bhabha scattering in collinear kinematics

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Abstract

The process of large–angle high energy electron–positron scattering with emission of one hard photon almost collinear to one of the charged particles momenta is considered. The differential cross section with radiative corrections due to emission of virtual and soft real photons calculated to a power accuracy is presented. Emission of two hard photons and total expressions for radiative correction are given in leading logarithmical approximation. The latter are illustrated by numeric estimates. A relation of the results with structure function formalism is discussed.

PACS: 12.20.–m Quantum electrodynamics, 12.20.Ds Specific calculations [1]

1 Introduction

The process of electron–positron scattering is commonly used for luminosity measurements at e^+e^- colliders. It has almost pure electrodynamical nature and could therefore be described to any desired precision within a framework of perturbative QED. Nevertheless the accuracy of modern experiments is ahead of that provided by theory. A lot of work has recently been done to uplift the theoretical uncertainty to about one per mille under conditions of small–angle Bhabha scattering at LEP1 [1] and afterwards up to $0.05 - 0.06\%$[2].

The large–angle kinematics of Bhabha scattering process is extensively used for calibration purposes at e^+e^- colliders of moderately high energies, such as ϕ, J/ψ, B, and c/τ factories and LEP2. At the Born and one–loop levels the process was investigated in detail in [3, 4, 5, 6, 7], taking into account both QED and electroweak effects.

In paper [8] we considered Bhabha scattering to $\mathcal{O}(\alpha)$ order exactly improved by the structure function method. The latter, based on the renormalization group approach, allows to evaluate the leading radiative corrections to higher orders, including all the terms $\sim (\alpha L_s)^n$, $n = 2, 3, \ldots$, where $L_s = \ln(s/m^2)$ is a large logarithm, s is the total center-of-mass (cms) energy of incoming particles squared and m is the mass of fermion.

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[1] Preprint Bicocca-FT-99-13 and hep-ph/9905331

To reach the one per mille accuracy it is required to take into account radiative corrections (RC) up to third order within the leading logarithmic approximation (LLA) and up to second order in the next–to–leading approximation (NLA). In a series of papers several sources of these corrections were considered in detail [9, 10, 11, 12].

In a recent publication [12] the contribution due to virtual and soft photon corrections to large–angle radiative Bhabha scattering was calculated for the general case of hard photon emission at large angle with respect to all charged particles momenta. In the present work we are going to consider the complementary specific kinematics, in which the photon moves within a narrow cone of small opening angle $\theta_0 \ll 1$ together with one of the incoming or outgoing charged particles. Thus, the result obtained here may be used in experiments with the tagging of scattered electron (positron) in detectors of small aperture $\theta_0 \ll 1$.

Our paper is organized as follows. In Sec. 2 the Born level cross section of radiative Bhabha scattering is revised in the collinear kinematics of photon emission along initial (scattered) electron. We introduce here the physical gauge of real photon that is extensively used in the next sections. In Sec. 3 a set of crossing transformations which enables us to consider in some detail only the scattering type amplitudes of loop corrections to the process is described. Besides, we restrict ourselves to the kinematics of hard photon emission along initial electron. In Sec. 4 the corrections due to virtual and soft real photon emission in the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ are considered. The general expression for correction in the case of hard photon emission along scattered electron is given in Sec. 5. In Sec. 6 we consider a contribution (in LLA) coming from two hard photon emission and derive a general expression for radiative correction. In conclusion we discuss the relation with structure function approach and the accuracy of the results obtained. Some useful expressions for loop integrals are given in the Appendix and the results of numeric estimates are given in graphs.

2 Born expressions in collinear kinematics

Let us begin revising the radiative Bhabha scattering process

$$e^-(p_1) \ + \ e^+(p_2) \ \rightarrow \ e^-(p_1') \ + \ e^+(p_2') \ + \ \gamma(k_1) \tag{1}$$

at the tree level. We define the collinear kinematical domains as those in which the hard photon is emitted close (within a narrow cone with opening angle $\theta_0 \ll 1$) to the incident ($\theta_{1(2)} = \widehat{\boldsymbol{p}_{1(2)}\boldsymbol{k}_1} < \theta_0$) or the outgoing electron (positron) ($\theta'_{1(2)} = \widehat{\boldsymbol{p}'_{1(2)}\boldsymbol{k}_1} < \theta_0$) direction of motion. Because of the symmetry between electron and positron we may restrict ourselves to a consideration of only two collinear regions, which correspond to the emission of the photon along the electron momenta. The two remaining contributions to the differential cross section of the process (1) can be obtained by the substitution $\mathcal{Q}$

$$\mathrm{d}\sigma_{\mathrm{coll}} \ = \ \left[1 + \mathcal{Q}\begin{pmatrix} p_1 \leftrightarrow p_2 \\ p_1' \leftrightarrow p_2' \end{pmatrix}\right]\Big\{\mathrm{d}\sigma^{\gamma}(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1) + \mathrm{d}\sigma^{\gamma}(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1')\Big\}. \tag{2}$$

To begin with, let us recall the known expression [13] in Born approximation for the general kinematics, i.e. assuming all the squares of the momenta transferred among fermions to be large compared to the electron mass squared:

$$\mathrm{d}\sigma_0^{\gamma} \ = \ \frac{\alpha^3}{8\pi^2 s}T\mathrm{d}\Gamma, \qquad \mathrm{d}\Gamma = \frac{\mathrm{d}^3\boldsymbol{p}_1'\mathrm{d}^3\boldsymbol{p}_2'\mathrm{d}^3\boldsymbol{k}_1}{\varepsilon_1'\varepsilon_2'\omega_1}\delta^4(p_1 + p_2 - p_1' - p_2' - k_1), \tag{3}$$

$$
\begin{aligned}
T &= \frac{S}{tt_1ss_1}\left[ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)\right] \\
&- \frac{16m^2}{\chi_2'^2}\left(\frac{s}{t_1}+\frac{t_1}{s}+1\right)^2 - \frac{16m^2}{\chi_1'^2}\left(\frac{s}{t}+\frac{t}{s}+1\right)^2 - \frac{16m^2}{\chi_2^2}\left(\frac{s_1}{t_1}+\frac{t_1}{s_1}+1\right)^2 \\
&- \frac{16m^2}{\chi_1^2}\left(\frac{s_1}{t}+\frac{t}{s_1}+1\right)^2, \\
S &= 4\left[\frac{s}{\chi_1\chi_2}+\frac{s_1}{\chi_1'\chi_2'}-\frac{t_1}{\chi_1\chi_1'}-\frac{t}{\chi_2\chi_2'}+\frac{u_1}{\chi_2\chi_1'}+\frac{u}{\chi_1\chi_2'}\right], \\
& s=(p_1+p_2)^2, \quad s_1=(p_1'+p_2')^2, \quad t=(p_2-p_2')^2, \quad t_1=(p_1-p_1')^2, \\
& u=(p_1-p_2')^2, \quad u_1=(p_2-p_1')^2, \quad \chi_i=2p_ik_1, \quad \chi_{1,2}'=2p_{1,2}'k_1.
\end{aligned}
$$

In the collinear kinematical domain in which $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ the above formula takes the form

$$
\begin{aligned}
\mathrm{d}\sigma_0^\gamma(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1) &= \frac{\alpha^3}{\pi^2 s}\frac{\mathrm{d}^3\boldsymbol{k}_1}{\omega_1}\frac{1}{\chi_1}\Upsilon F\frac{\mathrm{d}^3\boldsymbol{p}_1'\mathrm{d}^3\boldsymbol{p}_2'}{\varepsilon_1'\varepsilon_2'}\delta^4((1-x)p_1+p_2-p_1'-p_2') \\
&= \mathrm{d}W_{p_1}\mathrm{d}\sigma_0((1-x)p_1,p_2), \\
\Upsilon &= \frac{1+(1-x)^2}{x(1-x)}-\frac{2m^2}{\chi_1}, \qquad F=\left(\frac{s_1}{t}+\frac{t}{s_1}+1\right)^2,
\end{aligned}
\tag{4}
$$

where

$$
\begin{aligned}
& s_1=s(1-x), \quad y_1=\frac{\varepsilon_1'}{\varepsilon}=2\frac{1-x}{a}, \quad y_2=\frac{\varepsilon_2'}{\varepsilon}=\frac{2-2x+x^2+cx(2-x)}{a}, \\
& a=2-x+cx, \quad \omega_1=\varepsilon x, \quad s=4\varepsilon^2, \quad \chi_1=\frac{s}{2}x(1-c_1\beta), \quad \beta=\sqrt{1-\frac{m^2}{\varepsilon^2}}, \\
& t=t_1(1-x)=-s\frac{(1-x)^2(1-c)}{a}, \quad c=\cos(\widehat{\boldsymbol{p}_1\boldsymbol{p}_1'}), \quad c_1=\cos(\widehat{\boldsymbol{p}_1\boldsymbol{k}_1}), \\
& \mathrm{d}W_{p_1}=\frac{\alpha}{2\pi^2}\frac{1-x}{\chi_1}\Upsilon\frac{\mathrm{d}^3\boldsymbol{k}_1}{\omega_1}.
\end{aligned}
\tag{5}
$$

Here y_i are the energy fractions of the scattered leptons and $\mathrm{d}\sigma_0(p_1(1-x),p_2)$ is the cross section of the elastic Bhabha scattering process.

Throughout the paper we use the following relations among invariants

$$
s_1+t+u_1=4m^2-\chi_1\approx 0, \qquad s+t_1+u=4m^2+\chi_1\approx 0.
$$

In the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$ we have

$$
\begin{aligned}
\mathrm{d}\sigma_0^\gamma(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1') &= \frac{\alpha}{2\pi^2}\frac{1}{\chi_1'}\tilde{\Upsilon}\frac{\mathrm{d}^3\boldsymbol{k}_1}{\omega_1}(1-x)\mathrm{d}\sigma_0(p_1,p_2), \\
\tilde{\Upsilon} &= \frac{1+(1-x)^2}{x}-\frac{2m^2}{\chi_1'}.
\end{aligned}
\tag{6}
$$

These expressions could also be inferred by using the method of quasi-real electrons [14] and starting from the non-radiative Bhabha cross section.

After integration over a hard collinear ($\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$) photon angular phase space, the cross section of radiative Bhabha scattering in the Born approximation is found to be

$$\begin{aligned} \left.\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\right|_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1} &= \frac{4\alpha^3}{s}\left[\frac{1+(1-x)^2}{x}L_0 - 2\frac{1-x}{x}\right] \\ &\times \left(\frac{3-3x+x^2+2cx(2-x)+c^2(1-x(1-x))}{(1-x)(1-c)a^2}\right)^2(1+\mathcal{O}(\theta_0^2)), \end{aligned} \tag{7}$$

where $L_0 = \ln(\varepsilon\theta_0/m)^2$. And in the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$ it reads

$$\begin{aligned} \left.\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\right|_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1'} &= \frac{\alpha^3}{4s}\left[\frac{1+(1-x)^2}{x}L_0' - 2\frac{1-x}{x}\right]\left(\frac{3+c^2}{1-c}\right)^2(1+\mathcal{O}(\theta_0^2)), \\ L_0' &= \ln\left(\frac{\varepsilon_1'\theta_0}{m}\right)^2, \qquad \varepsilon_1' = \varepsilon(1-x). \end{aligned} \tag{8}$$

The simplest way to reproduce these results is to use the physical gauge for the real photon which in the beam cms sets the photon polarization vector to be a space-like 3-vector $\boldsymbol{e}_\lambda$ having density matrix

$$\sum_\lambda e_\mu^\lambda e_\nu^{\lambda *} = \left\{ \begin{array}{ll} 0, & \text{if } \mu \text{ or } \nu = 0 \\ \delta_{\mu\nu} - n_\mu n_\nu, & \mu=\nu=1,2,3 \end{array} \right. , \qquad \boldsymbol{n} = \frac{\boldsymbol{k}_1}{\omega_1},$$

with the properties

$$\begin{aligned} \sum_\lambda |e_\lambda|^2 &= -2, \quad \sum_\lambda |p_1 e_\lambda|^2 = \varepsilon^2(1-c_1^2), \\ \sum_\lambda |p_1' e_\lambda|^2 &= \frac{t_1 u_1}{s}, \quad \sum_\lambda (p_1 e_\lambda)(p_1' e_\lambda)^* \overset{\theta\to 0}{\sim} \theta. \end{aligned} \tag{9}$$

These properties enable us to omit mass terms in the calculations of traces and, besides, to restrict ourselves to the consideration of *singular* terms (see Eq. (10)) only, both at the Born and one–loop level. As shown in [15], this gauge is proved useful for a description of jet production in quantum chromodynamics; it is also very well suited to our case because it allows to simplify a lot the calculation with respect, for instance, to the Feynman gauge. What is more, it possesses another very attractive feature related with the structure of the correction to be mentioned below (see Appendix).

With these tools at our disposal let us turn now to the main point. The contributions, which survive the limit $\theta_0 \to 0$, arise from the terms containing

$$\frac{(p_1 e)^2}{\chi_1^2}, \quad \frac{e^2}{\chi_1}, \quad \frac{(p_1' e)^2}{\chi_1}. \tag{10}$$

Other omitted terms (in particular those which do not contain a factor χ_1^{-1}) can be safely neglected since they give a contribution of the order of θ_0^2 which determines the accuracy of our calculations

$$1 + \mathcal{O}\left(\frac{\alpha}{\pi}\theta_0^2 L_s\right), \qquad \frac{m}{\varepsilon} \ll \theta_0 \ll 1. \tag{11}$$

In the realistic case this corresponds to an accuracy of the order of per mille.

3 Crossing relations

In this and the next section we shall consider the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$. In the case of photon emission along p_1' one can get the desired expression by using the *left-to-right* permutation

$$|M|^2_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1'} = \mathcal{Q}\begin{pmatrix} p_1 \leftrightarrow -p_1' \\ p_2 \leftrightarrow -p_2' \end{pmatrix} |M|^2_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1}. \tag{12}$$

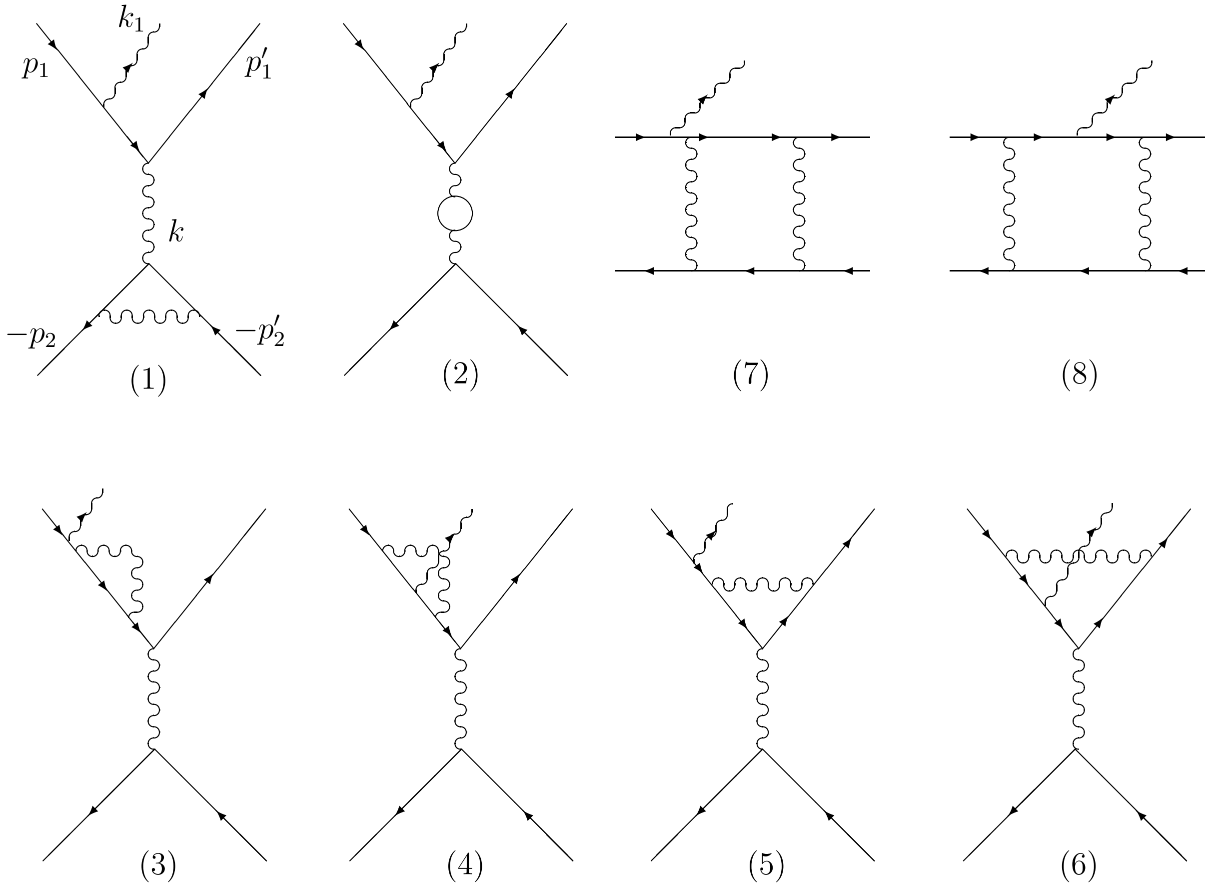

Figure 1: Some representatives of FD for radiative Bhabha scattering up to second order:*(1)* is the vertex insertion; *(2)* is the vacuum polarization insertion; graphs denoted by *(3)*,*(4)* are of the L-type, *(5)* is of G_1-type, *(6)* is of G_2-type, *(7)* is of B-type and *(8)* is of P-type.

From now on we deal with scattering type amplitudes (FD) with the emission of hard photon by initial electron. This is possible due to the properties of the physical gauge. The contribution of annihilation type amplitudes may be derived by applying the momenta replacement operation as follows:

$$\Delta|M|^2_{\text{annihilation}} = \{\mathcal{Q}(p_1' \leftrightarrow -p_2)\}\, \Delta|M|^2_{\text{scattering}} \equiv \{Q_1\}\, \Delta|M|^2_{\text{scattering}}. \tag{13}$$

In considering FD with two photons in the scattering channel (box FD) one may examine only those with uncrossed photons because a contribution of the others may be obtained by the permutation $p_2 \leftrightarrow -p_2'$. Thus the general answer becomes

$$|M|^2_{\boldsymbol{k}_1\|\boldsymbol{p}_1} = \Re e\{(1+Q_1)[G+L] + \frac{1}{s_1 t}(1+Q_1)(1+Q_2)[s_1 t(B+P)]\}, \tag{14}$$

with the permutation operators acting as

$$Q_1 F(s_1,t_1,s,t) = F(t,s,t_1,s_1), \quad Q_2 F(s,u,s_1,u_1) = F(u,s,u_1,s_1).$$

4 Virtual and soft photon emission in $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ kinematics

One–loop QED RC (which are described by seventy two Feynman diagrams) can be classified out into the two gauge invariant subsets (see Fig.1):

- single photon exchange between electron and positron lines (G,L–type);
- double photon exchange between electron and positron lines (B,P–type).

For L-type FD (see Fig. 1(3,4)) the initial spinor $u(p_1)$ is replaced by $(\alpha/(2\pi))A_2\hat{k}_1\hat{e}u(p_1)$, with

$$A_2 = \frac{1}{\chi_1}\left\{-\frac{\rho}{2(\rho-1)} + \frac{2\rho^2-3\rho+2}{2(\rho-1)^2}L_\rho + \frac{1}{\rho}\left[-\mathrm{Li}_2(1-\rho) + \frac{\pi^2}{6}\right]\right\},$$
$$L_\rho = \ln\rho, \qquad \rho = \frac{\chi_1}{m^2}.$$

The relevant contribution to the matrix element squared and summed over spin states reads

$$\Delta|M|^2_L = 2^9\pi^2\alpha^4\frac{A_2}{\chi_1}\frac{s_1^3-u_1^3}{s_1 t^2}\left[Y - \frac{2(2-x)}{1-x}W\right], \tag{15}$$
$$Y = 4(p_1 e)^2 - \frac{x}{1-x}e^2\chi_1, \qquad W = (p_1 e)^2.$$

The contribution of vertex insertion, vacuum polarization[2] and G_1-type (see Fig. 1(1,2,5)) has the following form

$$\begin{aligned}
\Delta|M|^2_{\Pi,\Gamma,\Gamma_a} &= 2^{10}\pi^2\alpha^4\left[\Pi_t + \Gamma_t + \frac{1}{4}\Gamma_a\right]\frac{s_1^3-u_1^3}{t^2 s_1\chi_1^2}Y, \qquad (16)\\
\Pi_t &= \frac{1}{3}L_t - \frac{5}{9}, \quad \Gamma_t = (L_\lambda - 1)(1-L_t) - \frac{1}{4}L_t - \frac{1}{4}L_t^2 + \frac{\pi^2}{12},\\
\Gamma_a &= -3L_t^2 + 4L_tL_\rho + 3L_t + 4L_\lambda - 2\ln(1-\rho) - \frac{\pi^2}{3} + 2\mathrm{Li}_2(1-\rho) - 4,\\
L_\lambda &= \ln\frac{m}{\lambda}, \quad L_t = \ln\frac{-t}{m^2}, \quad \mathrm{Li}_2(z) = -\int_0^z\frac{\mathrm{d}x}{x}\ln(1-x).
\end{aligned}$$

Here λ is as usual the IR cut-off parameter to be cancelled at the end of calculus against a soft photon contribution.

[2]For realistic applications one should also add to Π the contributions due to μ and τ leptons and hadrons.

For the contribution of G_2-type FD (see Fig. 1(6)) with four denominators we obtain

$$\Delta|M|^2_G = 2^9\alpha^4\pi^2\frac{s_1^3-u_1^3}{ts_1\chi_1(1-x)}\left[(J-J_1)Y+\frac{2(2-x)}{1-x}W(J_{11}-J_1+xJ_{1k}-xJ_k)\right]. \quad (17)$$

It turns out that only the scalar integral and the coefficients before p_1, k_1 in the vector and tensor integrals give non-vanishing contribution in the limit $\theta_0 \to 0$

$$\int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{(1,k^\mu,k^\mu k^\nu)}{(0)(1)(2)(q)} = (J, J_1p_1^\mu+J_kk_1^\mu, J_{11}p_1^\mu p_1^\nu + J_{kk}k_1^\mu k_1^\nu + J_{1k}(p_1k_1)^{\mu\nu}),$$
$$(0)=k^2-\lambda^2,\ (1)=k^2-2p_1k,\ (2)=k^2-2p_1'k,\ (q)=k^2-2k(p_1-k_1)-\chi_1,$$
$$(ab)^{\mu\nu}=a^\mu b^\nu+a^\nu b^\mu,$$

and the terms having no p_1 or k_1 momentum in the decomposition have been omitted for their unimportance.

The B-type FD shown in Fig. 1(7) with uncrossed legs gives

$$\Delta|M|^2_B = 2^9\pi^2\alpha^4Y\frac{1}{s_1t\chi_1^2}\Bigg[(u_1^3-s_1^3)s_1(B+a-b)-u_1^3s_1\left(c+a_{1'2'}+a_{1'2}+\frac{2}{s_1}a_g\right)$$
$$+ s_1^3(c[t-u_1]+2J_0)\Bigg], \quad (18)$$

where the coefficients are associated with scalar, vector and tensor integrals over loop momentum

$$\int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{(1,k^\mu,k^\mu k^\nu)}{b_0b_1b_2b_3} = (B,B^\mu,B^{\mu\nu}), \qquad J_0=\int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{1}{b_1b_2b_3},$$
$$b_0 = k^2-\lambda^2, \quad b_1=k^2+2p_1'k, \quad b_2=k^2-2p_2'k, \quad b_3=k^2-2qk+t, \quad q=p_2'-p_2,$$
$$B^\mu = (ap_1'+bp_2'+cp_2)^\mu,$$
$$B^{\mu\nu} = a_gg^{\mu\nu}+a_{1'1'}p_1^{'\mu}p_1^{'\nu}+a_{22}p_2^\mu p_2^\nu+a_{2'2'}p_2^{'\mu}p_2^{'\nu}+a_{1'2}(p_1'p_2)^{\mu\nu}+a_{1'2'}(p_1'p_2')^{\mu\nu}$$
$$+ a_{22'}(p_2p_2')^{\mu\nu}.$$

For P-type FD (see Fig. 1(8)) with uncrossed photon legs we have

$$\Delta|M|^2_P = 2^9\pi^2\alpha^4\frac{s_1^3-u_1^3}{t\chi_1(1-x)}\left[Y(E-E_1)+\frac{2(2-x)}{1-x}W(E_{11}-E_1+xE_{1k}-xE_k)\right]. \quad (19)$$

Here we are using the definition (with tensor structures giving no contributions in the limit $\theta_0 \to 0$ dropped)

$$\int\frac{\mathrm{d}^4k}{\mathrm{i}\pi^2}\frac{(1,k^\mu,k^\mu k^\nu)}{a_0a_1a_2a_3a_4} = (E, E_1p_1^\mu+E_kk_1^\mu, E_{11}p_1^\mu p_1^\nu+E_{kk}k_1^\mu k_1^\nu+E_{1k}(p_1^\mu k_1^\nu+p_1^\nu k_1^\mu)),$$
$$a_0 = k^2-\lambda^2, \qquad a_1=k^2-2p_1k, \qquad a_2=k^2-2k(p_1-k_1)-\chi_1,$$
$$a_3 = k^2+2p_2k, \qquad a_4=k^2-2qk+t.$$

Note that in the evaluating of P-type FD we are allowed to put $k_1=xp_1$, thus keeping only p_1 momentum containing terms in the decomposition.

Collecting all the contributions (for the explicit expressions of all the coefficients see Appendix) given above we arrive at the general expression for the virtual corrections with $\rho = x[1 + (\varepsilon\theta/m)^2] \ll s/m^2$

$$
\begin{aligned}
2\Re e \sum (M_0^* M)_{\boldsymbol{k}_1 \| \boldsymbol{p}_1} &= \frac{2^{11}\alpha^4\pi^2}{\chi_1} F\Upsilon \Big\{ \frac{2-x}{1-x}\frac{w}{\Upsilon}\Phi + 2L_\lambda(2 - L_t - L_{t_1} - L_s \\
&- L_{s_1} + L_u + L_{u_1}) + \frac{\pi^2}{3} + \mathrm{Li}_2(x) - \frac{101}{18} + \ln\left|\frac{\rho}{1-\rho}\right| + L_{u_1}^2 - L_t^2 \\
&- L_{s_1}^2 + L_\rho \ln(1-x) + \frac{11}{3}L_t - \vartheta + \ln^2\frac{s_1}{t} + \frac{1}{F}\Big[\Pi + 3\frac{t^3 - u_1^3}{s_1^2 t}\ln\frac{s_1}{t} \\
&+ \frac{2u_1(u_1^2 + s_1^2) - ts_1^2}{4t^2 s_1}\ln^2\frac{u_1}{t} + \frac{2u_1(u_1^2 + t^2) - t^2 s_1}{4ts_1^2}\ln^2\frac{u_1}{s_1} \\
&+ \frac{s_1}{2t}\ln\frac{u_1}{t} + \frac{t}{2s_1}\ln\frac{u_1}{s_1} - \frac{3}{4}\pi^2\left(\frac{s_1}{t} + \frac{t}{s_1}\right)\Big]\Big\},
\end{aligned}
\tag{20}
$$

where we have used the following definitions

$$
\begin{aligned}
\vartheta &= \frac{x}{\rho - x}\Big[\mathrm{Li}_2(1-\rho) - \frac{\pi^2}{6} + \mathrm{Li}_2(x) + L_\rho \ln(1-x)\Big], \\
\Pi &= \frac{s_1^3 - u_1^3}{s_1 t^2}\Big[\frac{\pi}{\alpha}\left(\frac{1}{1-\Pi_t} - 1\right) - \frac{1}{3}L_t + \frac{5}{9}\Big] + \frac{t^3 - u_1^3}{s_1^2 t}\Big[\frac{\pi}{\alpha}\Re e\left(\frac{1}{1-\Pi_{s_1}} - 1\right) - \frac{1}{3}L_{s_1} + \frac{5}{9}\Big], \\
\Pi_{s_1} &= \frac{1}{3}(L_{s_1} - \mathrm{i}\pi) - \frac{5}{9}, \quad \Phi = \chi_1 A_2 + t_1\chi_1(J_{11} - J_1 + xJ_{1k} - xJ_k), \quad w = \frac{1}{x} - \frac{1}{\rho}, \\
L_{s_1} &= \ln\frac{s_1}{m^2}, \quad L_u = \ln\frac{-u}{m^2}, \quad L_{u_1} = \ln\frac{-u_1}{m^2}, \quad L_t = \ln\frac{-t}{m^2}, \quad L_{t_1} = \ln\frac{-t_1}{m^2}.
\end{aligned}
$$

After integration over χ_1 one gets additional large logs of the form $L_0 = L_s + \ln(\theta_0^2/4)$. Terms containing the last factor have to be cancelled against a contribution coming from the emission of hard photon outside a narrow cone $\theta < \theta_0 \ll 1$ (and supplied by the same set of virtual and soft corrections), which was considered in [12]. In the case of two hard photon emission it is necessary to consider four kinematical regions, namely when both are emitted inside/outside a cone and one inside/another outside.

Fortunately enough, the w-structure, which obviously violates factorization feature, does not contribute in LLA due to a cancellation of large logs in Φ. What for a correction to the above structure coming from P-type graph it vanishes in the sum of FD with crossed and uncrossed photon legs (for a more comprehensive account see Appendix).

The total expression can be obtained by summing virtual photon emission corrections and those arising from the emission of additional soft photon with energy exceeding no $\Delta\varepsilon \ll \varepsilon$.

The emission of a soft photon is seen as a process factored out of a hard subprocess (in our case the latter is exactly a hard collinear photon emission) so this is seemingly come into an evident conflict with a hard collinear emission. Nevertheless, arguments similar to those given in the paper devoted to the problem of DIS with tagged photon [16] may be applied in the present paper: the factorization of the two in the *differential* cross section is present and we are, hence, allowed to consider a soft photon emission restricted as usual by

$$
\frac{\Delta\varepsilon}{\varepsilon} \ll 1. \tag{21}
$$

Thus the soft correction can be written as

$$\sum |M|^2_{\rm hard+soft} = \sum |M|^2_B w_{\rm soft}(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1), \tag{22}$$
$$w_{\rm soft}(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1) = -\frac{\alpha}{4\pi^2}\int\limits_{\omega<\Delta\varepsilon} \frac{{\rm d}^3\boldsymbol{k}}{\sqrt{\boldsymbol{k}^2+\lambda^2}}\left(-\frac{p_1}{p_1k}+\frac{p_1'}{p_1'k}+\frac{p_2}{p_2k}-\frac{p_2'}{p_2'k}\right)^2,$$

where M_B denotes the matrix element of the hard photon emission at the Born level and in the kinematics $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ it reads

$$\sum |M|^2_B = \frac{2^{11}\alpha^3\pi^3}{\chi_1}\Upsilon F. \tag{23}$$

Now let us check the cancellation of the terms containing L_λ. Indeed it takes place in the sum of contributions arising from emission of virtual and soft real photons. To show that we bring the soft correction into the form

$$\begin{aligned}
w_{\rm soft}(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1) &= \frac{\alpha}{\pi}\Big\{2\left(\ln\frac{\Delta\varepsilon}{\varepsilon}+L_\lambda\right)(-2+L_s+L_{s_1}+L_t+L_{t_1}-L_u-L_{u_1})+\frac{1}{2}(L_s^2+L_{s_1}^2 \\
&+ L_t^2+L_{t_1}^2-L_u^2-L_{u_1}^2)+\ln y_1(L_{u_1}-L_{s_1}-L_{t_1})+\ln y_2(L_u-L_t-L_{s_1}) \\
&+ \ln(y_1y_2)-\frac{2\pi^2}{3}-\frac{1}{2}\ln^2\frac{y_1}{y_2}+{\rm Li}_2\left(\frac{1+c_{1'2'}}{2}\right)+{\rm Li}_2\left(\frac{1+c_{1'}}{2}\right) \\
&+ {\rm Li}_2\left(\frac{1-c_{2'}}{2}\right)-{\rm Li}_2\left(\frac{1-c_{1'}}{2}\right)-{\rm Li}_2\left(\frac{1+c_{2'}}{2}\right)\Big\},
\end{aligned} \tag{24}$$

where c_i are the cosines of emission angles of i-th particle with respect to the beam direction ($\boldsymbol{p}_1$ in cms), $c_{1'2'}$ is the cosine of the angle between scattered fermions in cms of the colliding particles and y_i are their energy fractions and in the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ we have

$$c_1' = c, \qquad \frac{1+c_{1'2'}}{2}=1-\frac{1-x}{y_1y_2}, \qquad \frac{1-c_2'}{2}=\frac{y_1(1+c)}{2y_2(1-x)}. \tag{25}$$

Then the cancellation of infrared singularities in the sum is evident from comparison of Eqs. (20,24). The terms with $\ln(\Delta\varepsilon/\varepsilon)$ should be cancelled when adding a contribution of a second hard photon having energy above the registration threshold $\Delta\varepsilon$.

The complete expression for the correction in the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ reads

$$\begin{aligned}
R &= 2\Re e\sum(M_0^*M)+|M|^2_{\rm soft}=\frac{2^{11}\alpha^4\pi^2}{\chi_1}F\Upsilon\Big\{\frac{2-x}{1-x}\frac{w}{\Upsilon}\Phi+4\ln\left(\frac{\Delta\varepsilon}{\varepsilon}\right)\Big[-1+L_{t_1} \\
&+\frac{1}{2}\left(-\ln(1-x)+2\ln\frac{s}{-u}\right)\Big]+\frac{11}{3}L_t+(L_\rho-L_t)\ln(1-x)-L_t\ln(y_1y_2)+\ln^2\frac{s_1}{-t} \\
&+\ln y_1\ln(1-x)+\ln(y_1y_2)\left(1+\ln\frac{-u}{s}\right)-\frac{\pi^2}{3}+{\rm Li}_2(x)-\frac{101}{18}-\vartheta+\ln\left|\frac{\rho}{1-\rho}\right| \\
&-\frac{1}{2}\ln^2\frac{y_1}{y_2}+\ln(1-x)\ln\frac{-u}{s}+{\rm Li}_2\left(\frac{1+c_{1'2'}}{2}\right)+{\rm Li}_2\left(\frac{1+c_{1'}}{2}\right)+{\rm Li}_2\left(\frac{1-c_{2'}}{2}\right) \\
&-{\rm Li}_2\left(\frac{1-c_{1'}}{2}\right)-{\rm Li}_2\left(\frac{1+c_{2'}}{2}\right)+\frac{1}{F}\Big[\Pi+3\frac{t^3-u_1^3}{s_1^2t}\ln\frac{s_1}{-t}+\frac{2u_1(u_1^2+s_1^2)-ts_1^2}{4t^2s_1}\ln^2\frac{u_1}{t} \\
&+\frac{2u_1(u_1^2+t^2)-t^2s_1}{4ts_1^2}\ln^2\frac{-u}{s}+\frac{s_1}{2t}\ln\frac{u_1}{t}+\frac{t}{2s_1}\ln\frac{-u}{s}-\frac{3}{4}\pi^2\left(\frac{s_1}{t}+\frac{t}{s_1}\right)\Big]\Big\},
\end{aligned} \tag{26}$$

$${\rm d}\sigma(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1)=\frac{1}{2^{11}\pi^5 s}R{\rm d}\Gamma.$$

5 Kinematics $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$

We put here a set of replacements one can use in order to obtain the modulus of matrix element squared and summed over spin states for the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$, starting from the analogous expression for $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ (Eq. (14)) and using the replacement of momenta $p_1 \leftrightarrow -p_1', p_2 \leftrightarrow -p_2'$. The last operation results in the following substitutions:

$$\begin{aligned} x &\rightarrow -\frac{x}{1-x}, \\ \chi_1 &\rightarrow -\chi_1', \\ s &\leftrightarrow s_1, \\ u &\leftrightarrow u_1, \\ t \rightarrow t \quad &, \quad t_1 \rightarrow t_1. \end{aligned} \tag{27}$$

Then under these permutations the expression for virtual corrections given in Eq. (20) gets transformed yielding the following result for the collinear domain $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$

$$\begin{aligned} 2\Re e \sum (M_0^* M)_{\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'} &= \frac{2^{11}\alpha^4\pi^2}{\chi_1'} \tilde{F}\tilde{\Upsilon}\Bigg\{ \frac{2-x}{1-x}\frac{\tilde{w}}{\tilde{\Upsilon}}\tilde{\Phi} + 2L_\lambda(2 - L_t - L_{t_1} - L_s \\ &- L_{s_1} + L_u + L_{u_1}) + \frac{\pi^2}{3} + \mathrm{Li}_2\left(\frac{-x}{1-x}\right) - \frac{101}{18} + \ln\left(\frac{\xi}{\xi+1}\right) + L_u^2 \\ &- L_t^2 - L_s^2 - L_\xi \ln(1-x) + \frac{11}{3}L_t + \ln^2\frac{s}{-t} + \frac{1}{\tilde{F}}\Bigg[\Pi + 3\frac{t^3-u^3}{s^2 t}\ln\frac{s}{-t} \\ &+ \frac{2u(u^2+s^2) - ts^2}{4t^2 s}\ln^2\frac{u}{t} + \frac{2u(u^2+t^2) - t^2 s}{4ts^2}\ln^2\frac{-u}{s} + \frac{s}{2t}\ln\frac{u}{t} - \tilde{\vartheta} \\ &+ \frac{t}{2s}\ln\frac{-u}{s} - \frac{3}{4}\pi^2\left(\frac{s}{t} + \frac{t}{s}\right)\Bigg]\Bigg\}, \end{aligned} \tag{28}$$

with

$$\begin{aligned} \tilde{\Pi} &= \frac{s^3-u^3}{st^2}\left[\frac{\pi}{\alpha}\left(\frac{1}{1-\Pi_t} - 1\right) - \frac{1}{3}L_t + \frac{5}{9}\right] + \frac{t^3-u^3}{s^2 t}\left[\frac{\pi}{\alpha}\Re e\left(\frac{1}{1-\Pi_s} - 1\right) - \frac{1}{3}L_s + \frac{5}{9}\right], \\ \tilde{F} &= \left(\frac{s}{t} + \frac{t}{s} + 1\right)^2, \qquad \tilde{w} = -\frac{1-x}{x} + \frac{1}{\xi}, \qquad \xi = \frac{\chi_1'}{m^2} \end{aligned}$$

and $\tilde{\Phi}, \tilde{\vartheta}$ derived upon applying a set of replacements from Eq. (27) on the quantities Φ, ϑ.

The contribution from the soft photon emission is described by

$$\begin{aligned} w_{\mathrm{soft}}(\boldsymbol{k}_1 \parallel \boldsymbol{p}_1') &= \frac{\alpha}{\pi}\Bigg[4(\ln\frac{\Delta\varepsilon}{\varepsilon} + L_\lambda)\left(-1 + L_s + \ln\frac{1-c}{1+c} + \frac{1}{2}\ln(1-x)\right) + L_s^2 + 2L_s\ln\frac{1-c}{1+c} \\ &- \frac{1}{2}\ln^2(1-x) + \ln(1-x) + \ln^2\frac{1-c}{2} - \ln^2\frac{1+c}{2} - \frac{2\pi^2}{3} \\ &+ 2\mathrm{Li}_2\left(\frac{1+c}{2}\right) - 2\mathrm{Li}_2\left(\frac{1-c}{2}\right)\Bigg]. \end{aligned} \tag{29}$$

The total correction for the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$ has the following form

$$\tilde{R} = 2\Re e\sum(M_0^* M) + |M|^2_{\mathrm{soft}} = \frac{2^{11}\alpha^4\pi^2}{\chi_1'}\tilde{F}\tilde{\Upsilon}\Bigg\{\frac{2-x}{1-x}\frac{\tilde{w}}{\tilde{\Upsilon}}\tilde{\Phi} + 4\ln\left(\frac{\Delta\varepsilon}{\varepsilon}\right)\Bigg(-1 + L_s$$

$$
\begin{aligned}
&+\ \frac{1}{2}\ln(1-x)+\ln\frac{1-c}{1+c}\Big)+\frac{\pi^2}{3}+\mathrm{Li}_2\left(\frac{-x}{1-x}\right)-\frac{101}{18}+\ln\left(\frac{\xi}{\xi+1}\right)-2\ln^2(1-x)\\
&+\ \frac{11}{3}L_t-L_\xi\ln(1-x)+\ln^2\frac{s}{-t}-\frac{2}{3}\pi^2+\ln(1-x)-\tilde{\vartheta}+2\mathrm{Li}_2\left(\frac{1+c}{2}\right)\\
&-\ 2\mathrm{Li}_2\left(\frac{1-c}{2}\right)+\frac{1}{\tilde{F}}\Big[\tilde{\Pi}+3\frac{t^3-u^3}{s^2t}\ln\frac{s}{-t}+\frac{1}{4t^2s}\ln^2\left(\frac{u}{t}\right)(2u(u^2+s^2)-ts^2)\\
&+\ \frac{1}{4ts^2}\ln^2\left(\frac{-u}{s}\right)(2u(u^2+t^2)-t^2s)+\frac{s}{2t}\ln\frac{u}{t}+\frac{t}{2s}\ln\frac{-u}{s}-\frac{3}{4}\pi^2\left(\frac{s}{t}+\frac{t}{s}\right)\Big]\Big\}, \qquad (30)\\
&\mathrm{d}\sigma(\boldsymbol{k}_1\parallel\boldsymbol{p}_1')=\frac{1}{2^{11}\pi^5s}\tilde{R}\mathrm{d}\Gamma.
\end{aligned}
$$

Performing the integration over a hard photon angular phase space (inside a narrow cones) we put the RC to the cross section coming from virtual and soft real additional photons valid to a logarithmic accuracy in the form

$$
\frac{\mathrm{d}\sigma^{\gamma(V+S)}}{\mathrm{d}x\mathrm{d}c}=\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\frac{\alpha}{\pi}\left[C\frac{\Delta\varepsilon}{\varepsilon}+L_t\Xi_L+\Xi\right]. \qquad (31)
$$

In the Fig. 2(a,b) given are the ratio of $\Xi/(L_t\Xi_L)$ versus x for the collinear kinematics considered above.

6 Two hard photon emission and results in LLA

Turning to the structure of the result obtained, it should be noted that all the terms quadratic in *large* logarithms $L_{t_1}\sim L_{s_1}\sim L_u\gg L_\rho$ are mutually cancelled out as it should be.

From the formula (26) it immediately follows that (upon doing an integration over a hard photon angular (within a narrow cone) phase space) the w-term that is not proportional to Υ, which is in fact the kernel of the non-singlet electron structure function, is not dangerous in the sense of a feasible violation of the expected Drell–Yan form of the cross section, because it does contribute only at next-to-leading order.

Performing the above mentioned integration and confining ourselves to LLA we get for the sum of virtual and soft photons

$$
\frac{\mathrm{d}\sigma^{\gamma(S+V)}}{\mathrm{d}x\mathrm{d}c}=\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\frac{\alpha}{\pi}L\left[4\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{11}{3}-\frac{1}{2}\ln(1-x)-\ln(y_1y_2)\right]. \qquad (32)
$$

The LLA contribution coming from the emission of second hard photon with total energy exceeding $\Delta\varepsilon$ consists of a part corresponding to the case in which both hard photons (with total energy εx) are emitted by initial electron [9]

$$
\begin{aligned}
\frac{\mathrm{d}\sigma^{2\gamma}}{\mathrm{d}x\mathrm{d}c} &= \frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\frac{\alpha}{\pi}L\Big[\frac{x\mathcal{P}_\Theta^{(2)}(1-x)}{4(1+(1-x)^2)}+\frac{1}{2}\ln(1-x)-\ln\frac{\Delta\varepsilon}{\varepsilon}-\frac{3}{4}\Big], \qquad (33)\\
P_\Theta^{(2)}(z) &= 2\Big[\frac{1+z^2}{1-z}\Big(2\ln(1-z)-\ln z+\frac{3}{2}\Big)+\frac{1+z}{2}\ln z-1+z\Big],
\end{aligned}
$$

and the remaining part which describes the emission of second hard photon along scattered electron and positrons. The latter, upon combining with the part of contributions of soft and virtual photons to our process

$$\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\frac{3\alpha}{\pi}L\left[\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{3}{4}\right],$$

may be represented via electron structure function in the spirit of the Drell–Yan approach

$$\begin{aligned}\langle\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\rangle\Big|_{\boldsymbol{k}_1\|\boldsymbol{p}_1} &= \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}L_0\int \mathrm{d}z_2\mathrm{d}z_3\mathrm{d}z_4\mathcal{D}(z_2)\mathcal{D}(z_3)\mathcal{D}(z_4) \\ &\times \frac{\mathrm{d}\sigma_0(p_1(1-x),z_2p_2;q_1,q_2)}{\mathrm{d}c},\end{aligned} \tag{34}$$

with the non-singlet structure function $\mathcal{D}(z)$ [17]

$$\begin{aligned}\mathcal{D}(z) &= \delta(1-z)+\frac{\alpha}{2\pi}L\mathcal{P}^{(1)}(z)+\left(\frac{\alpha}{2\pi}L\right)^2\frac{1}{2!}\mathcal{P}^{(2)}(z)+\dots, \\ P^{(1,2)}(z) &= \lim_{\Delta\to0}\Big\{\delta(1-z)P_\Delta^{(1,2)}+\Theta(1-\Delta-z)P_\Theta^{(1,2)}(z)\Big\}, \\ P_\Delta^{(1)} &= 2\ln\Delta+\frac{3}{2},\quad P_\Theta^{(1)}(z)=\frac{1+z^2}{1-z},\quad P_\Delta^{(2)}=\left(2\ln\Delta+\frac{3}{2}\right)^2-\frac{2\pi^2}{3},\ \dots\end{aligned} \tag{35}$$

These functions describe the emission of (real and virtual) photons both by final electron and by positrons. The multiplier before the integral stands for the emission of an hard photon by the initial electron. Thus Eq. (34) actually represents the partially integrated Drell–Yan form of the cross section. Quite the same arguments are applicable to the second case in which an hard photon is emitted by the final electron.

The cross section of the hard sub-process $e(p_1z_1)+\bar{e}(p_2z_2)\to e(q_1)+\bar{e}(q_2)$ entering Eq. (34) has the form

$$\frac{\mathrm{d}\sigma_0(z_1p_1,z_2p_2;q_1,q_2)}{\mathrm{d}c}=\frac{8\pi\alpha^2}{s}\left[\frac{z_1^2+z_2^2+z_1z_2+2c(z_2^2-z_1^2)+c^2(z_1^2+z_2^2-z_1z_2)}{z_1(1-c)(z_1+z_2+c(z_2-z_1))^2}\right]^2. \tag{36}$$

The momenta of scattered electron q_1 and positron q_2 are completely determined by the energy-momentum conservation law

$$\begin{aligned}&q_1^0=\varepsilon\frac{2z_1z_2}{z_1+z_2+c(z_2-z_1)},\qquad q_1^0+q_2^0=\varepsilon(z_1+z_2),\\ &c=\cos\widehat{\boldsymbol{q}_1,\boldsymbol{p}}_1,\qquad z_1\sin\widehat{\boldsymbol{q}_1,\boldsymbol{p}}_1=z_2\sin\widehat{\boldsymbol{q}_2,\boldsymbol{p}}_1.\end{aligned}$$

In general their energies differ from those detected in experiment $\varepsilon_1',\varepsilon_2'$, namely

$$\varepsilon_1'=q_1^0z_3,\qquad \varepsilon_2'=q_2^0z_4,$$

whereas the emission angles are the same in LLA.

Collecting the two expressions presented in Eqs. (32,33) one can rewrite the result in LLA as

$$\frac{\mathrm{d}\sigma^\gamma}{\mathrm{d}x\mathrm{d}c}\Big|_{\boldsymbol{k}_1\|\boldsymbol{p}_1}=\left(\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\right)_{\boldsymbol{k}_1\|\boldsymbol{p}_1}\{1+\delta_1\},$$

$$\delta_1=\left(\frac{\langle\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\rangle}{\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}}\right)_{\boldsymbol{k}_1\|\boldsymbol{p}_1}-1+\frac{\alpha}{\pi}L\left[\frac{2}{3}-\ln(y_1y_2)+\frac{x\mathcal{P}_\Theta^{(2)}(1-x)}{4(1+(1-x)^2)}\right]. \tag{37}$$

For the case $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$ the correction is found to be

$$
\begin{aligned}
\left.\frac{\mathrm{d}\sigma^\gamma}{\mathrm{d}x\mathrm{d}c}\right|_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1'} &= \left(\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\right)_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1'}\{1+\delta_{1'}\}\,, \\
\delta_{1'} &= \left(\frac{\langle\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\rangle}{\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}}\right)_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1'} - 1 + \frac{\alpha}{\pi}L\left[\frac{2}{3} + \frac{x\mathcal{P}_\Theta^{(2)}(1-x)}{4(1+(1-x)^2)}\right], \\
\left.\langle\frac{\mathrm{d}\sigma_0^\gamma}{\mathrm{d}x\mathrm{d}c}\rangle\right|_{\boldsymbol{k}_1\parallel\boldsymbol{p}_1'} &= \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}L_0'\int \mathrm{d}z_1\mathrm{d}z_2\mathrm{d}z_4\mathcal{D}(z_1)\mathcal{D}(z_2)\mathcal{D}(z_4) \\
&\times \frac{\mathrm{d}\sigma_0(z_1p_1,z_2p_2;q_1,q_2)}{\mathrm{d}c}\,,
\end{aligned}
\tag{38}
$$

with $L_0' = L_0 + 2\ln(1-x)$.

For the case when the energies of scattered fermions are not detected the expressions (34,38) may be simplified due to $\int \mathrm{d}z\mathcal{D}(z) = 1$ and z_3, z_4–independence of the integrand in $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ kinematics (z_4–independence in $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$ case).

The x–dependence of δ_1 are shown in the Fig. 3 for different values of the cosine of scattering angle c. For a hard photon emission by final particles the correction δ_1' strongly depends on the experimental conditions of particles detection: the energy thresholds of detection of scattered fermions. This dependence for δ_1 is much more weaker, namely about 1%.

In conclusion let us recapitulate the results given in Eqs. (37,38). They both respect the Drell–Yan form for a cross section in LLA. Nevertheless a certain deviation away from RG structure function representation at a second order of PT in $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$ kinematics is observed. The term destroying expectations based on RG approach comes from definite contribution of a soft photon emission, the term with $\ln(y_1y_2)$ in Eq. (37) which cannot be included into the structure function approach. Its appearance is presumably a mere consequence of a complicate kinematics of $2 \to 3$ type hard subprocess (see [12]); for such a kind of processes the validity of the Drell–Yan form for a cross section was not proved so far. Another possible way out is a careful analysis of a *conflict* between a soft and hard collinear photon emission. We have used the factorized form of a soft photon emission (22) under the condition (21). But, to the moment, this representation in the peculiar case at hand is not rigorously proved as well.

The accuracy of our calculations of virtual and soft photon corrections is determined by the omitted terms of the order of

$$
1 + \mathcal{O}\left(\theta_0^2\frac{\alpha}{\pi}L_s, \frac{m^2}{s}\frac{\alpha}{\pi}L_s\right), \tag{39}
$$

which corresponds to a per mille level. The accuracy of the correction coming from two hard photon emission is determined by $\mathcal{O}((\alpha/\pi)\ln(4/\theta_0^2))$ and at 1% level.

Acknowledgments

We thank A.B. Arbuzov for a critical reading of the manuscript, many valuable comments and participating at the early stage of the investigation. We are also indebted to L. Lipatov for many discussions elucidating factorization issues. One of us (EAK) is grateful to the physical department of Insubria University (Como) for a warm hospitality during accomplishment of the final part of this work and to the Landau Network-Centro Volta grant for financial support. The support of EAK by INTAS grant 93-1867 ext. and of EAK and BGS by Russian Foundation for Basic Research grant 99-02-17730 is acknowledged.

Appendix

Here we give the expressions for the quantities associated with G-type integrals:

$$\begin{aligned}
J &= -\frac{1}{\chi_1 t_1}\left[-2L_\lambda L_{t_1} + 2L_{t_1}L_\rho - L_t^2 - 2\mathrm{Li}_2(x) - \frac{\pi^2}{6}\right], \\
J_1 &= \frac{1}{t_1\chi_1}\int\limits_0^\rho \frac{\mathrm{d}z}{1-z}\frac{\ln z}{1-\lambda z} = \frac{A}{t_1\chi_1}\left(1+\frac{x}{\rho-x}\right) = \frac{A+\vartheta}{t_1\chi_1}, \\
J_k &= -\frac{1}{t_1\chi_1\rho}\int\limits_0^\rho \frac{\mathrm{d}z}{1-z}\frac{z\ln z}{1-\lambda z}, \\
J_{11} &= -\frac{1}{t_1\chi_1}\int\limits_0^\rho \frac{\mathrm{d}z}{(1-z)(1-\lambda z)}\left(1+\frac{z\ln z}{1-z}\right), \qquad (A.1)\\
J_{1k} &= \frac{1}{t_1\chi_1\rho}\int\limits_0^\rho \frac{z\mathrm{d}z}{(1-z)(1-\lambda z)}\left(1+\frac{z\ln z}{1-z}\right), \\
A &= \mathrm{Li}_2(1-\rho) - \frac{\pi^2}{6} + \mathrm{Li}_2(x) + L_\rho \ln(1-x), \quad \lambda = \frac{x}{\rho}, \quad \rho = \frac{\chi_1}{m^2}.
\end{aligned}$$

In the limit $\rho \gg 1$ we have

$$\Phi = \chi_1 A_2 + t_1\chi_1(J_{11} - J_1 + xJ_{1k} - xJ_k) = -\frac{1}{2} + \mathcal{O}(\rho^{-1})$$

and that is the reason why w-structure does contribute only to next-to-leading terms.

In general the expression for 5-denominator one–loop scalar, vector and tensor integrals are some complicate functions of five independent kinematical invariants (in the derivation we extensively use the technique developed in [18]). In the limit $m^2 \ll \chi_1 \ll s \sim -t$ they may be considerably simplified because of singular $1/\chi_1$ terms only kept:

$$\begin{aligned}
E &= \frac{1}{s_1}D_{0124} + \frac{1}{t}D_{0123}, \\
E_1 &= -xE_k = \frac{1}{2\chi_1}\left(D_{0134} - (1-x)D_{0234} - xD_{1234} + \chi_1 E\right), \\
D_{0124} &= \frac{1}{xt_1\chi_1}\left[L_\rho^2 + 2L_\rho \ln\frac{x}{1-x} - \ln^2\frac{x}{1-x} - \frac{2\pi^2}{3}\right], \qquad (A.2)\\
\Re e D_{0123} &= \frac{1}{s\chi_1}\left[L_{s_1}^2 - 2L_{s_1}L_\rho - 2L_s L_\lambda + \frac{\pi^2}{6} + 2\mathrm{Li}_2(x)\right], \\
\Re e D_{0234} &= \frac{1}{s_1 t}\left[L_{s_1}^2 + 2L_{s_1}L_\lambda - 2L_\rho L_{s_1} + 2L_{s_1}L_t - \frac{5\pi^2}{6}\right],
\end{aligned}$$

$$\Re e D_{0134} = \frac{1}{st}\left[L_s^2 + 2L_sL_\lambda - 2(L_{t_1}+\ln(x))L_s + 2L_sL_t + \frac{7\pi^2}{6}\right],$$
$$\Re e D_{1234} = -\frac{1}{s_1xt_1}\left[-L_s^2 + 2L_s(L_{t_1}+\ln(x)) + 2L_{s_1}L_\lambda - \frac{7\pi^2}{6}\right].$$

The structure $E_{11}+xE_{1k}$ has the form $1/(s\chi_1)f(x,\chi_1)$ and will vanish after performing the operation $(1+Q_2)s_1tP$ given in (14) which yields a contribution of P-type graphs with crossed and uncrossed photon legs.

The following coefficient for the scalar integral is obtained in the calculation of B-type FD:

$$B = \frac{1}{s_1t}\left[L_{s_1}^2 + 2L_{s_1}L_\lambda - 2L_{s_1}L_\rho + 2L_{s_1}L_t + \frac{\pi^2}{6}\right]. \tag{A.3}$$

For the vector integral coefficients we get

$$\begin{aligned}
a &= -\frac{1}{2s_1u_1t}\left[-\pi^2s_1 + 2u_1\mathrm{Li}_2(1-\rho) - s_1L_t^2 + tL_{s_1}^2 - 2tL_{s_1}L_t\right],\\
b &= -\frac{1}{2s_1t}\left[\frac{2\pi^2}{3} + 2\mathrm{Li}_2(1-\rho) - 2L_{s_1}^2 + 4L_{s_1}L_\rho - 2L_{s_1}L_t\right],\\
c &= \frac{1}{2s_1u_1t}\left[2u_1\mathrm{Li}_2(1-\rho) + \frac{\pi^2}{6}(4u_1+6t) + (t-2u_1)L_{s_1}^2 - s_1L_t^2 + 4u_1L_{s_1}L_\rho + 2s_1L_{s_1}L_t\right].
\end{aligned} \tag{A.4}$$

The relevant quantities for tensor B-type integrals are:

$$\begin{aligned}
a_{1'2'} &= \frac{1}{s_1t}\left(\frac{\rho}{\rho-1}L_\rho - L_t\right), \qquad a_g = -\frac{1}{4u_1}[(L_{s_1}-L_t)^2+\pi^2],\\
a_{1'2} &= -\frac{1}{2u_1^2}[(L_t-L_{s_1})^2+\pi^2] + \frac{1}{tu_1}(L_{s_1}-L_t) - \frac{1}{s_1t}\left(\frac{\rho}{\rho-1}L_\rho - L_{s_1}\right),\\
J_0 &= \frac{1}{s_1}\left[\frac{3}{2}L_{s_1}^2 - 2L_{s_1}L_\rho - \mathrm{Li}_2(1-\rho) - \frac{4\pi^2}{3}\right].
\end{aligned} \tag{A.5}$$

As has been mentioned in the text, the physical gauge exploited provides a direct extraction of the kernel of the structure function out of the traces both in the tree- and loop-level amplitudes. The pattern emerging

$$\begin{aligned}
(\hat{p}_1-\hat{k}_1+m)\hat{e}(\hat{p}_1+m)\hat{e}(\hat{p}_1-\hat{k}_1+m) &= 4(p_1e)^2(\hat{p}_1-\hat{k}_1) - e^2\chi_1\hat{k}_1\\
&\approx (1-x)Y\hat{p}_1,\\
\hat{k}_1\hat{e}(\hat{p}_1+m)\hat{e}(\hat{p}_1-\hat{k}_1+m) &\approx (1-x)\left(2\frac{2-x}{1-x}W - Y\right)\hat{p}_1
\end{aligned} \tag{A.6}$$

shows this clearly.

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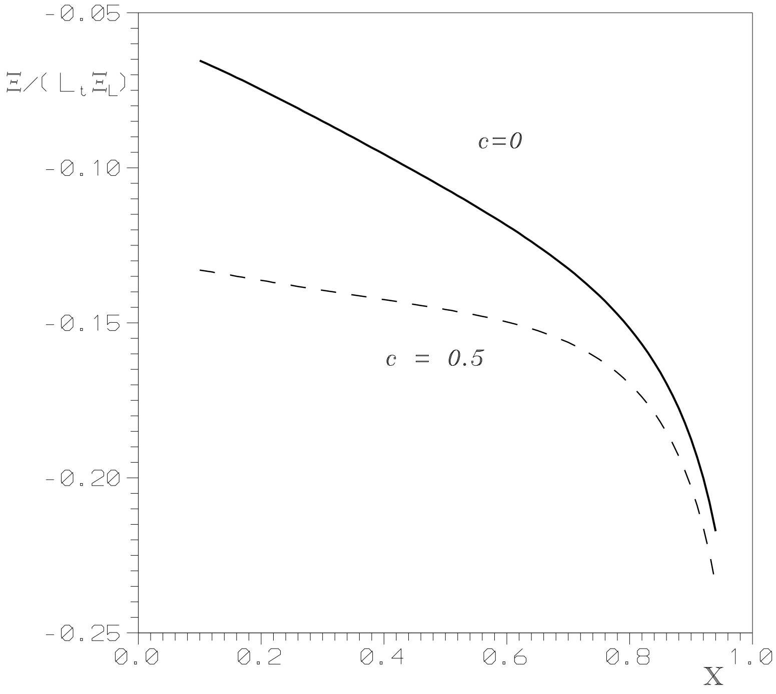
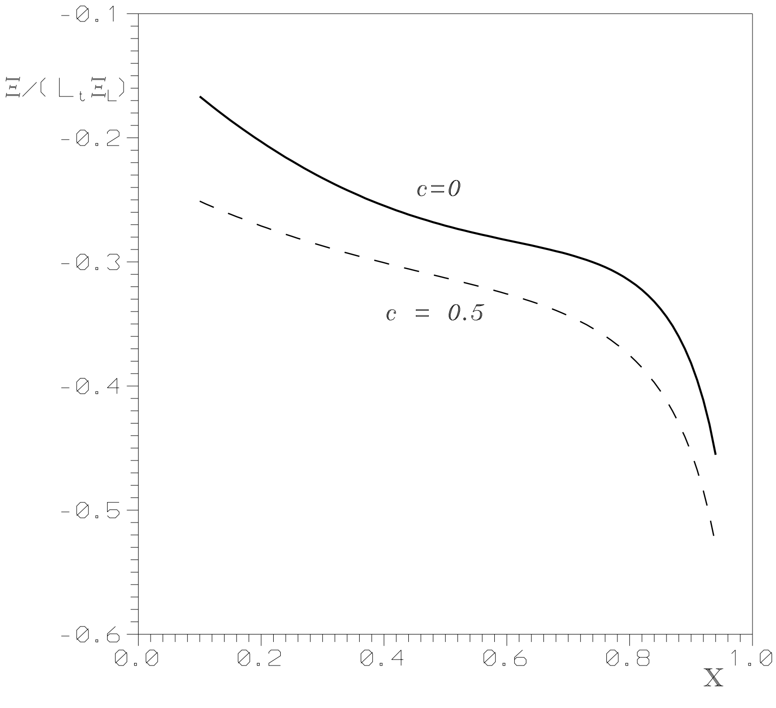
Figure captions

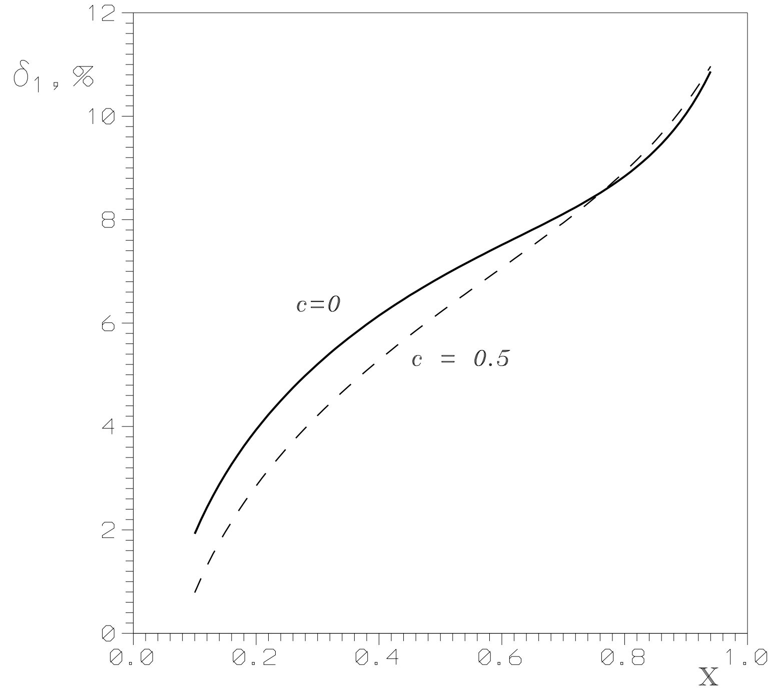
Figure 2.
The ratio $\frac{\Xi}{L_t \Xi_L}$ (see Eq. (31)) versus $x = \frac{\omega_1}{\varepsilon}$ for the case:
a) $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1$.
b) $\boldsymbol{k}_1 \parallel \boldsymbol{p}_1'$.

Figure 3.
The x–dependence of δ_1 (see Eq. (37))
for different values of the cosine of scattering angle c.

Other parameters chosen are: $\theta_0 = 0.1,\ \varepsilon = 1$ GeV.


-0.05
-0.10
-0.15
-0.20
-0.25
0.0
0.2
0.4
0.6
0.8
1.0
x
c=0
c = 0.5


-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0.0
0.2
0.4
0.6
0.8
1.0
x
c=0
c = 0.5



δ₁, %
c=0
c = 0.5
X